

On the Impact of Transposition Errors in Diffusion-Based Channels

Werner Haselmayr[†], *Member, IEEE*, Syed Muhammad Haider Aejaz[†], *Member, IEEE*,
A. Taufiq Asyhari, *Member, IEEE*, Andreas Springer, *Member, IEEE*, and Weisi Guo*, *Member, IEEE*

Abstract—Transposition errors fundamentally undermine reliability and capacity in molecular communication, when individual particles are used for information encoding. Recently, several channel coding techniques have been proposed for mitigating the transposition effect. The presented techniques show promising results for diffusion-based channels with drift. However, so far no performance evaluation has been carried out for diffusion-based channels without drift, although in this case, transpositions are more prominent. In this work, we first derive an analytical expression for the uncoded bit error probability due to transpositions. Then, we compare the uncoded and coded error performance over diffusion-based channels with and without drift by means of computer simulations. The results reveal that for diffusion-based channels without drift a transmission with reasonable reliability can only be achieved by introducing a guard time between the codewords. This research lays the foundation for future development of strategies to mitigate transpositions in pure diffusion-based channels.

Index Terms—Guard time, Lévy distribution, molecular communication, inverse Gaussian distribution, transposition/crossover effect

I. INTRODUCTION

Molecular communication broadly defines the transmission of information using biochemical molecules over multiple distance scales [1]. Within multi-cellular organisms, molecular communication within cells, between local cells, and across the body of the organism (e.g., hormones) is essential for coordinated cellular action-reaction. Between organisms, molecular communication takes place over several kilometers distance in air and under water (e.g., pheromones), and is used to signal intent, assist navigation, and warn of impending dangers [2]. The aforementioned molecular communication largely relies on messenger molecules to transverse channels using some form of normal or anomalous diffusion mechanism, potentially combining microscopic discrete random walk with macroscopic continuum fluid mechanics.

Due to the potential for ultra-high energy efficiency [3], device dimension scalability, and bio-compatibility, molecular communication via diffusion (MCvD) has gathered intense research interest in academia [4]–[7] and also industry (see IEEE standardization efforts under P1906.1-2015 [8]). The majority of MCvD research and research projects have been

split between: i) fundamental understanding of molecular signaling channels [9]–[11], ii) exploiting the benefits of molecular signaling in nano-medicine applications [5], [12]–[14], and iii) developing and testing physical layer transmission schemes [15]–[18] and signaling protocols [19] for reliable multi-scale molecular signaling.

A. Review on the Transposition Effect

In MCvD, with or without drift, the information can be encoded in the number of particles, the release time of particles and the type of particles [7]. Due to the stochastic nature of diffusion-based channels, it may occur that a sequence of transmitted particles will arrive out-of-order at the receiver, i.e. particles that are released earlier arrive late – yielding transpositions¹ of bits or symbols [20]. Transpositions fundamentally limit the performance of time- and type-based modulation schemes using individual particles. If the information is encoded in the number of particles, transpositions have almost no effect, since the arrival time averaged over all particles is unlikely to vary significantly. Similarly, transmitting multiple particles instead of a single particle to convey time- or type-based information reduces the transposition effect [21], [22]. However, if nano-machines are only able to release a small number of particles due to molecular reservoir capacity issues or when nano-machines are moving [23], the receiver cannot rely on the law of large numbers to average out stochastic errors. Thus, in these scenarios, it is important to consider the effect of transpositions.

The implementation of an optimal detector for time- and type-based modulation is infeasible, even for a small number of particles, since all possible permutations of the particles arrival must be taken into account [22], [24]. A sub-optimal detector for time-based modulation was proposed in [24], which unfortunately is not applicable to type-based modulation. In recent years, several block coding techniques have been proposed for mitigating the transposition effect [15], [25], [26]. The code design is no longer based on the Hamming distance, which is only useful if the bits are corrupted by noise and is likely to be ineffective in the case of transpositions. Hence, other attributes, like for example the Hamming weight [15] or a molecular coding distance [25] were considered as suitable coding design paradigms. However, so far the error performance of the codes was only evaluated for diffusion-based channels with drift, although transpositions are more prominent in diffusion-based channels without drift. This is

Manuscript received January 9, 2017; revised Month Day, 2017. [†]The first two authors contributed equally to this work. *Corresponding author. W. Haselmayr, S. M. H. Aejaz, A. Springer are with the Johannes Kepler University Linz, Austria, A. T. Asyhari is with the University of Bradford, United Kingdom and W. Guo is with the School of Engineering, University of Warwick, United Kingdom.

¹In [20], transpositions are referred to as crossovers.

because the tail of the first hitting time distribution for pure diffusion-based channels is heavier compared to flow-induced channels [22], [27].

B. Contributions & Organization

Previous studies only provided analytical expressions for the transposition probability of two particles [26], [28]. In this work we provide an analytical expression for the probability of each permutation that can occur when a sequence of particles is released. Moreover, we derive the bit errors caused by each permutation.

To the best of our knowledge, the block coding schemes proposed for combating transposition errors have been only evaluated for diffusion-based channels with drift (e.g., [15] provides a comparative performance analysis). Since the transposition effect is more prominent in diffusion-based channels without drift, it is of high interest whether the coding schemes provide a reasonable error performance in such a scenario. Thus, in this work we provide an extensive error performance analysis for pure diffusion-based channels and a comparative study with flow-induced diffusion-based channels by means of computer simulations. The results show, that for pure diffusion-based channels a reasonable performance can only be achieved by introducing a guard time between the codewords. The rest of this paper is organized as follows: The system model is presented in Section II and the uncoded bit error probability due to transpositions is derived in Section III. The most promising block coding techniques for mitigating transposition errors are discussed in Section IV. Simulation results are presented in Section V and Section VI provides concluding remarks.

II. SYSTEM MODEL

We consider the transmission of L information bits $\mathbf{b} = [b_1, \dots, b_L]^T$ over a diffusion-based channel with and without drift. The bit sequence is divided into Q blocks of length² $k = L/Q$. If an (n, k) block code is applied, each k -bit block is mapped to an n -bit codeword, i.e. $\mathbf{c}_q = [c_{q,1}, \dots, c_{q,n}]^T$, $q = 1, \dots, Q$. Thus, in the coded case $M = nQ$ coded bits are transmitted. In the uncoded case the number of transmitted bits is $M = kQ = L$, since $n = k$. As modulation scheme, we use binary molecule shift keying (MoSK) [7], [29], which employs two different particles for information encoding. We employ type- A and type- B particles to encode bit 0 and bit 1, respectively. The m th particle is released by the transmitter into the medium at time

$$X_m = (m-1)T_b, \quad m = 1, \dots, M, \quad (1)$$

where T_b denotes the bit interval. The released particles propagate according to Brownian motion, with or without drift and independently of each other, to the receiver. The receiver is capable of detecting the two types of particles and particles arriving at the receiver are absorbed and removed from the environment. Due to the stochastic nature of the channel, the

particles arrive at the receiver at random time. The arrival time of the m th particle is given by [22], [27]

$$Y_m = X_m + Z_m, \quad m = 1, \dots, M, \quad (2)$$

where Z_m denotes the random propagation time until the first arrival, which is referred to as first hitting time.

The first hitting time of a diffusion-based channel without drift follows a Lévy distribution, with its probability density function (PDF) given by [27]

$$f_Z^{\mathcal{L}}(z) = \sqrt{\frac{c}{2\pi z^3}} \exp\left(-\frac{c}{2z}\right), \quad z > 0, \quad (3)$$

and its cumulative distribution function (CDF) given by

$$F_Z^{\mathcal{L}}(z) = \text{erfc}\left(\sqrt{\frac{c}{2z}}\right), \quad z > 0, \quad (4)$$

with the scale parameter $c = d^2/(2D_p)$. The distance between transmitter and receiver is denoted by d and D_p is the diffusion coefficient of the transmitted particles. The complementary error function $\text{erfc}(x)$ is defined by $\text{erfc}(x) = 2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$. The first hitting time for a diffusion-based channel with drift follows an inverse Gaussian distribution with its PDF given by [22]

$$f_Z^{\mathcal{IG}}(z) = \sqrt{\frac{\lambda}{2\pi z^3}} \exp\left(-\lambda \frac{(z-\mu)^2}{2\mu^2 z}\right), \quad z > 0, \quad (5)$$

and its CDF given by

$$F_Z^{\mathcal{IG}}(z) = \phi\left(\sqrt{\frac{\lambda}{z}}\left(\frac{z}{\mu} - 1\right)\right) + \exp\left(\frac{2\lambda}{\mu}\right) \phi\left(-\sqrt{\frac{\lambda}{z}}\left(\frac{z}{\mu} + 1\right)\right), \quad z > 0, \quad (6)$$

with mean $\mu = d/v$, the shape parameter $\lambda = d^2/(2D_p)$ and the standard normal distribution $\phi(x) = \exp(-x^2/2)/\sqrt{2\pi}$. The positive drift velocity from the transmitter to the receiver is denoted by v . Note that in the case of no drift ($v = 0$ m/s) the mean μ tends to infinity and the inverse Gaussian distribution turns into a Lévy distribution, i.e. $\lim_{\mu \rightarrow \infty} f_Z^{\mathcal{IG}}(z) = f_Z^{\mathcal{L}}(z)$. The receiver collects the particles based on their arrival time and treats them as bit 0 and bit 1 if type- A and type- B are received, respectively. If coding is applied at the transmitter, a proper decoding scheme is applied to obtain the information bits (cf. Section IV).

III. TRANSPOSITION EFFECT

Due to the random arrival time of the particles, they may arrive out-of-order and, thus, transpositions occur [20]. According to [26], we define that a level- l transposition occurs when two particles are released exactly l bit intervals apart, but their receiving order changes. For example, suppose that three particles are released in the order (p_1, p_2, p_3) and are received in the order (p_2, p_1, p_3) , which is illustrated in Fig. 1. In this case a level-1 transposition happened. If coding is applied at the transmitter, we can distinguish between two cases of transpositions:

²Without loss of generality, we assume that $k = L/Q \in \mathbb{N}$.

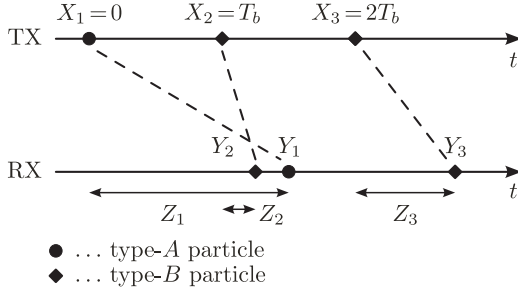


Fig. 1. Illustration of the transposition effect. Three particles are released in the order (p_1, p_2, p_3) and are received in the order (p_2, p_1, p_3) .

- 1) *Intra-codeword transpositions* refer to transpositions within a codeword.
- 2) *Inter-codeword transpositions* denote transpositions over different codewords.

In the uncoded case, only intra-codeword transpositions occur, since only a single block/codeword is transmitted. In the following, we provide an analytical expression for the uncoded bit error probability due to transpositions. The derivation can be divided into two steps. First, we determine the probability for each permutation that can occur when a sequence of M particles is released. Second, we provide an expression for the bit errors caused by each permutation. Let \mathcal{P} represent the set of all possible permutations on M released particles. The probability that a certain permutation $\pi \in \mathcal{P}$ arrives at the receiver is given by

$$P_{\text{perm}}(\pi) = \int_0^\infty \int_{-\infty}^{y_{\pi(M)}} \cdots \int_{-\infty}^{y_{\pi(2)}} \prod_{i=0}^{M-1} f_Z(y_{\pi(M-i)} - x_{\pi(M-i)}) dy_{\pi(1)} \cdots dy_{\pi(M)}, \quad (7)$$

where $f_Z(z)$ denotes the PDF of the Lévy distribution or the inverse Gaussian distribution given in (3) and (5), respectively. The release time of the m th particle is denoted by $x_m = (m-1)T_b$ (cf. (1)) and $\pi(i)$ denotes the i th element of the permutation π (e.g., $\pi(1) = 2$ for $\pi = (2, 1, 3)$). It is important to note that a permutation may not directly translate to an error. For example, consider three particles, each of either type-A or type-B, that are released in the order $(1, 2, 3) \triangleq (p_1, p_2, p_3)$. If the permutation $(2, 1, 3) \triangleq (p_2, p_1, p_3)$ arrives at the receiver, no error occurs when $(p_2 = A, p_1 = A, p_3 = B)$, but an error occurs if $(p_2 = B, p_1 = A, p_3 = B)$. The bit error probability for a certain permutation can be obtained as follows

$$P_b(\pi) = \frac{\text{disp}(\pi)}{2M}, \quad \pi \in \mathcal{P}, \quad (8)$$

where we denote the number of displacements of a permutation π by

$$\text{disp}(\pi) = \sum_{i=1}^M \left\lceil \frac{|\pi(i) - i|}{M} \right\rceil, \quad (9)$$

and the ceiling function $\lceil x \rceil$ denotes the mapping to the smallest following integer number. Combining (7) and (8) gives the uncoded bit error probability for a transmission of M bits

$$P_b = \sum_{\pi \in \mathcal{P}} P_{\text{perm}}(\pi) P_b(\pi). \quad (10)$$

It is important to note that the calculation of the bit error probability is only feasible if M is small, since \mathcal{P} has $M!$ elements. In the following example, we derive the bit error probability for the case when two particles are released and relate this expression to other definitions used in the literature.

Example 1. We consider an uncoded transmission of $L = M = 2$ bits using a single block, i.e. $Q = 1$ and $k = 2$. Thus, the set \mathcal{P} includes $M! = 2$ permutations π_0 and π_1 . The probability that the particles p_1 and p_2 arrive in the correct order, $\pi_0 = (1, 2) \triangleq (p_1, p_2)$, is given by (7)

$$P_{\text{perm}}(p_1, p_2) = \int_0^\infty \int_{-\infty}^{y_2} f_Z(y_2 - x_2) f_Z(y_1 - x_1) dy_1 dy_2 = \int_0^\infty f_Z(y_2 - x_2) \int_{-\infty}^{y_2} f_Z(y_1 - x_1) dy_1 dy_2. \quad (11)$$

Similarly, the probability that the particles p_1 and p_2 arrive out-of-order, $\pi_1 = (2, 1) \triangleq (p_2, p_1)$, can be computed by

$$P_{\text{perm}}(p_2, p_1) = \int_0^\infty f_Z(y_1 - x_1) \int_{-\infty}^{y_1} f_Z(y_2 - x_2) dy_2 dy_1. \quad (12)$$

By making the substitution $y_1 - x_1 = z_1$ and $y_2 - x_2 = z_2$ and let x_1 and x_2 be 0 and T_b , respectively (cf. (1) and (2)), we can rewrite (12) as follows

$$P_{\text{perm}}(p_2, p_1) = \int_0^\infty f_Z(z_1) \int_{-\infty}^{z_1 - T_b} f_Z(z_2) dz_2 dz_1 \quad (13)$$

$$= \int_0^\infty f_Z(z_1) F_Z(z_1 - T_b) dz_1, \quad (14)$$

where $F_Z(z)$ denotes the CDF of the Lévy distribution or the inverse Gaussian distribution given in (4) and (6), respectively. It is important to note that the expression in (14) corresponds to [26, Eq. (7)], which describes the probability that a transposition between two particles happens. Moreover, through substituting z_2 by $z_1 + z$ in (13) and by changing the order of the integrals results in

$$P_{\text{perm}}(p_2, p_1) = \int_{-\infty}^{-T_b} \int_0^\infty f_Z(z_1) f_Z(z + z_1) dz_1 dz = \int_{-\infty}^{-T_b} (f_Z * f_Z^-)(z) dz = \int_{-\infty}^{-T_b} f_{\Delta Z} dz = F_{\Delta Z}(-T_b), \quad (15)$$

TABLE I
CODEWORDS ASSIGNMENT FOR THE (4,2,1) ISI-FREE CODE.

Information bits	Assignment if the previous codeword has	
	last bit 0	last bit 1
00	0000	1111
01	0001	1000
10	0011	1100
11	0111	1110

with $f_Z^- = f_Z(-z)$ and $f_{\Delta Z}(z)$ and $F_{\Delta Z}(z)$ denote the PDF and the CDF of the random variable $\Delta Z = Z_2 - Z_1$, respectively. Thus, the integral on the r.h.s of (15) defines the probability $\Pr(\Delta Z < -T_b)$ which is equivalent to the probability that the particles p_1 and p_2 arrive out-of-order. It is important to note that for the Lévy distribution a closed-form expression for the PDF $f_{\Delta Z}$ was derived in [27, Eq. (19)]. However, no expression for the CDF $F_Z(-T_b)$ was provided in [27], but it can be either calculated numerically or by using tables of the standardized CDF.

According to (8), the bit error probability for the permutation $P_{perm}(p_2, p_1)$ is given by $P_b(p_2, p_1) = 1/2$ and, thus, the total bit error probability results in $P_b = 1/2 P_{perm}(p_2, p_1)$.

IV. TRANSPOSITION ERROR CORRECTION CODES

In recent years, several block coding techniques for mitigating the transposition effect have been introduced. In an (n, k) block code, a k -bit block is mapped to an n -bit codeword. Most existing block codes have been designed based on maximizing the minimum pairwise Hamming distance between the 2^k codewords chosen from 2^n possible codewords. However, this criterion is only useful if most of the bits are corrupted by noise and is likely to be ineffective when bits exchange their position. Thus, for the design of codes that mitigate transpositions, relevant attributes that are preserved after the permutation of the bit positions have to be identified. For the sake of completeness, we review the most promising block codes in the following.

A. Repetition Codes

Repetition codes repeat a particular bit n times. In the case of bit transpositions, the structure of a codeword is preserved and, thus, the code is effective for mitigating intra-codeword transpositions. However, repetition codes have a code rate of $R_{rep} = 1/n$ which becomes very small for a large n .

B. Distinct Hamming Weight Codes

Distinct Hamming weight (DHW) codes are based on the observation that the Hamming weight of a codeword is preserved when bits exchange their position [15]. Thus, such codes are effective for combating intra-codeword transpositions. The codebook of DHW codes consists of codewords with distinct Hamming weights. For example, the codebook of a (4,2) DHW code, with code rate 1/2, is given by $\mathcal{C}_{DHW} = \{0000, 1000, 1100, 1110\}$.

TABLE II
COMPARISON OF THE BER WITH THE BIT ERROR PROBABILITY P_b IN THE UNCODED CASE ($D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$, $T_b = 0.1 \text{ s}$).

n	P_b	BER
2	0.2462	0.2461
3	0.3292	0.3297
4	0.3670	0.3711

C. ISI-free Codes

A family of (n, k, l) ISI-free codes that address intra- and inter-codeword transpositions have been presented in [26], where l denotes the correctable transposition level. An example of a (4, 2, 1) ISI-free code, with code rate 1/2, is shown in Tab. I. It can be observed that, similar to DHW codes, intra-codeword transpositions are mitigated using codewords with distinct Hamming weights. Moreover, level- l transpositions between adjacent codewords are suppressed by ensuring that for two consecutive codewords, the l last bits of the first codeword are identical to the first l bits for the second codeword.

The decoding complexity of the aforementioned codes is very low, since it only involves counting the number of 1's. Analytical expressions that approximate the bit error probability of repetition and ISI-free codes were presented in [26]. The approximation uses the pairwise transposition probability defined in (12) and neglects transpositions that are larger than level-2. A comparative performance analysis of the aforementioned block codes was presented [15]. However, the performance was only evaluated for diffusion-based channels with drift. Thus, in the next section we provide a comprehensive performance analysis for pure diffusion-based channels and compare the results with flow-induced channels.

V. SIMULATION RESULTS

In this section, we present analytical and simulation results for the bit error probability in diffusion-based channels with and without drift. We consider uncoded as well as coded transmission and the simulation results were obtained using Monte-Carlo simulations.

A. Uncoded Case

Figs. 2 – 4 show the impact of the diffusion coefficient D_p , the distance d , and the bit interval T_b on the uncoded bit error rate (BER) for different block sizes n . In the uncoded case, we consider the transmission of a single block, i.e. $Q = L/n = 1$, and, thus, only intra-block/codeword transpositions occur. If not otherwise stated, we use $D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$ and $T_b = 0.1 \text{ s}$. Tab. II compares the BER results, obtained via simulations, with the bit error probability P_b , which was obtained by evaluating the analytical expression in (10). It shows that the simulation results match the analytical values very well.

To illustrate the impact of different diffusion coefficients, we chose $D_p \in \{600 \mu\text{m}^2/\text{s}, 4500 \mu\text{m}^2/\text{s}\}$, which corresponds to the diffusion coefficient of glucose and hydrogen in 25°C

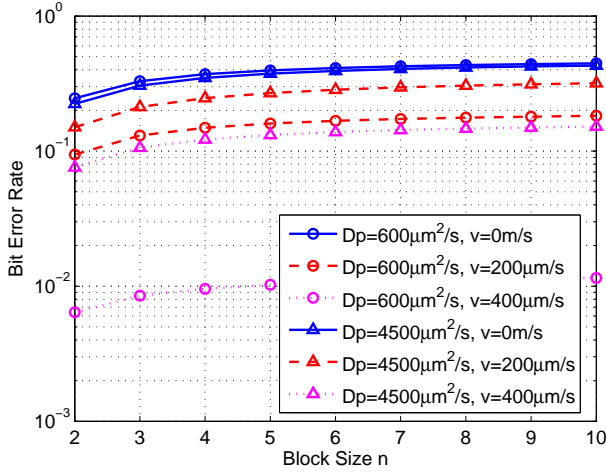


Fig. 2. Impact of the diffusion coefficient D_p on the bit error rate in the uncoded case ($d = 50 \mu\text{m}$ and $T_b = 0.1 \text{ s}$).

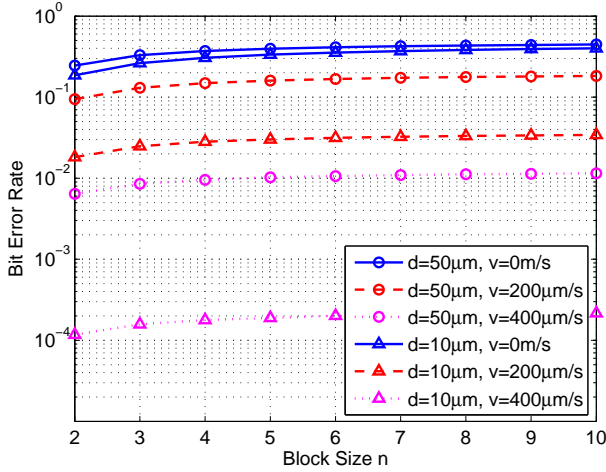


Fig. 3. Impact of the distance d on the bit error rate in the uncoded case ($D_p = 600 \mu\text{m}^2/\text{s}$ and $T_b = 0.1 \text{ s}$).

water, respectively³ [30]. It can be observed from Fig. 2 that the impact of different diffusion coefficients is negligible in the case of diffusion-based channels without drift. If there is a drift in the medium, particles with lower diffusion coefficient have a better error performance than particles with higher diffusion coefficients, since particles with lower diffusion coefficient experience a higher support due to drift.

Figs. 3 and 4 show the impact of different distances d and different bit intervals⁴ T_b . We chose $d \in \{10 \mu\text{m}, 50 \mu\text{m}\}$ and $T_b \in \{0.1 \text{ s}, 10 \text{ s}\}$. For diffusion-based channels without drift only a small gain can be observed, but for diffusion-based channels with drift the BER improves significantly when the

³Note that for binary MoSK two particles with almost equal D_p are required.

⁴We observed through simulations that the BER for $T_b = 10 \text{ s}$ with $v = 200 \mu\text{m/s}$ and $v = 400 \mu\text{m/s}$ lies far below 10^{-9} . However, we did not determine the exact values due to the very long simulation times and, thus, the corresponding BER curves are not shown in Fig. 4.

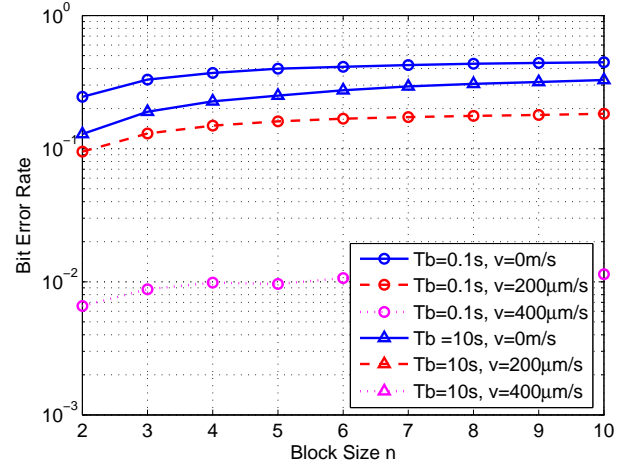


Fig. 4. Impact of the bit interval T_b on the bit error rate in the uncoded case ($D_p = 600 \mu\text{m}^2/\text{s}$ and $d = 50 \mu\text{m}$).

distance is reduced and the bit interval is increased.

It can be seen from Figs. 2 – 4 that an uncoded transmission with reasonable reliability (e.g., $\text{BER} < 10^{-4}$) is only possible for large bit intervals and high drift velocities. Moreover, the reliability can be further improved by reducing the distance d . Unfortunately, changing the distance is hardly possible and a high drift is often not available in the medium. Furthermore, increasing the bit interval reduces the bit rate. For this reason, different block coding techniques have recently been proposed, which we evaluate in the next section.

B. Coded Case

In this section, we evaluate different block coding techniques, namely repetition codes, ISI-free codes and DHW codes (cf. Section IV), to mitigate the transposition effect and, thus, to enable a reliable transmission even for low drift scenarios. In the coded case, we consider the transmission of L bits which are divided into $Q = L/n > 1$ blocks of size n and, thus, intra- and inter-codeword transpositions occur.

Fig. 5 shows the BER versus guard time T_G , which corresponds to the time between consecutive codewords. The aim for introducing a guard time is to reduce inter-codeword transpositions, since currently proposed codes can only tackle a limited number of transpositions between codewords. It can be observed that when no guard time is introduced, i.e. $T_G = T_b = 0.1 \text{ s}$, coding does not significantly improve the BER performance. However, for a guard time of $T_G > T_b$, a substantial BER improvement can be achieved. The performance improvement for diffusion-based channels with drift is much higher than for diffusion-based channels without drift. This is because the tail of the Lévy distribution is heavier compared to the inverse Gaussian distribution, which results in high level transpositions that are not limited to adjacent codewords. Thus, we expect that applying codes that can combat high level transpositions between adjacent codewords will not significantly boost the performance.

Fig. 6 shows the BER versus the guard time, which is chosen according to the percentile of particles arrivals, e.g., the 99th

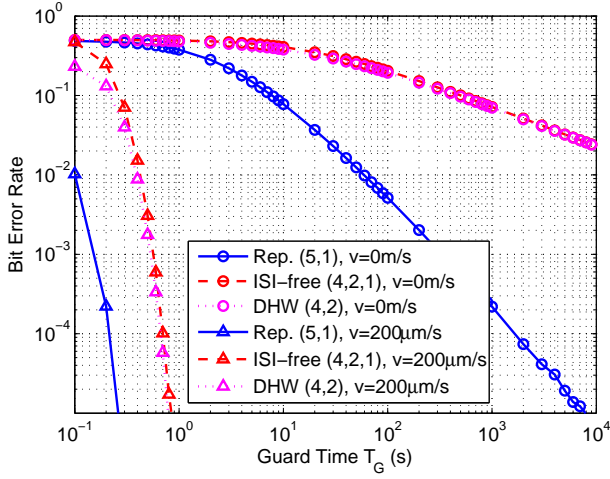


Fig. 5. Coded bit error rate versus guard time T_G ($D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$, $T_b = 0.1 \text{ s}$).

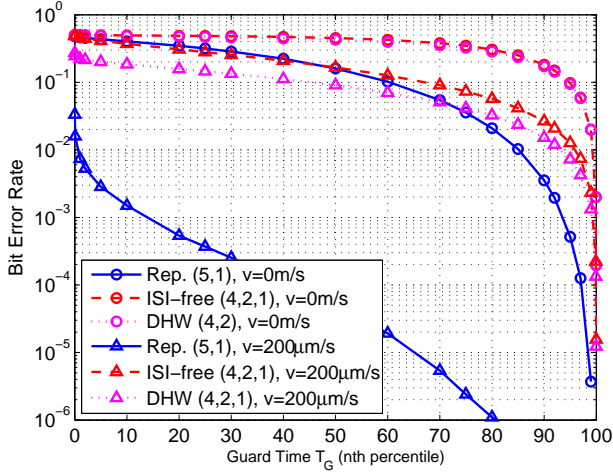


Fig. 6. Coded bit error rate versus percentile of particle arrivals ($D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$, $T_b = 0.1 \text{ s}$).

percentile corresponds to the guard time where 99% of the released particles are expected to be observed at the receiver. For diffusion-based channels without drift, a BER below 10^{-5} can be achieved with a repetition code at the 99th percentile. For flow-induced diffusion-based channels, a similar BER is achieved at the 75th percentile, but a BER below 10^{-7} is achieved at the 99th percentile (cf. Fig. 7). Please note that for pure diffusion-based channels, the 99th percentile corresponds to the guard time $T_G = 13262 \text{ s}$, whereas for diffusion-based channels with drift the 75th and 99th percentile corresponds to the guard time $T_G = 0.3 \text{ s}$ and $T_G = 0.52 \text{ s}$, respectively. Thus, to achieve a reasonable BER performance for diffusion-based channels without drift, almost all particles of a codeword must be received before the next is transmitted. The resulting guard time is very large due to the heavy tail of the Lévy distribution. For flow-induced diffusion-based channel, a significantly lower guard time is required to obtain comparable results.

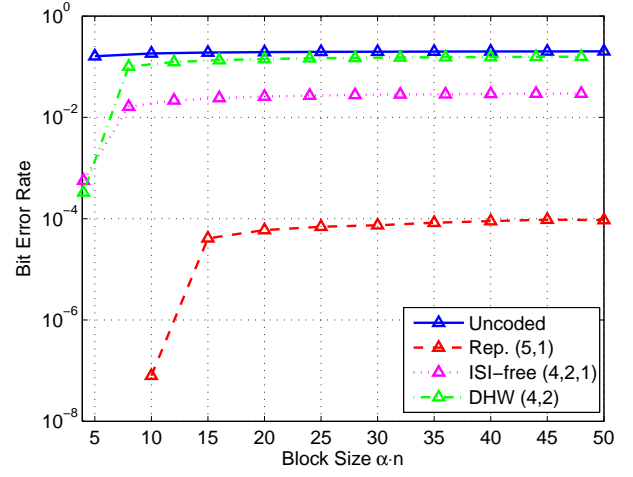


Fig. 7. Coded bit error rate versus block size for diffusion-based channels with drift ($D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$, $T_b = 0.1 \text{ s}$, $v = 200 \mu\text{m}/\text{s}$, 99th percentile).

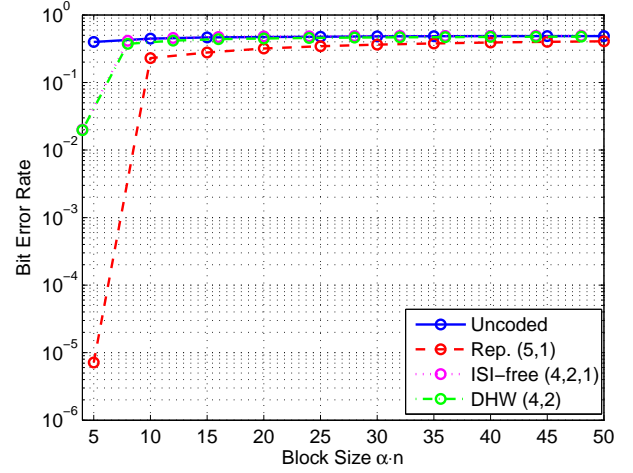


Fig. 8. Coded bit error rate versus block size for diffusion-based channels without drift ($D_p = 600 \mu\text{m}^2/\text{s}$, $d = 50 \mu\text{m}$, $T_b = 0.1 \text{ s}$, 99th percentile).

Figs. 7 and 8 show the BER when multiple codewords are transmitted consecutively, i.e. a guard time is only inserted between the concatenated codewords. The size of the resulting blocks is given by αn , where α denotes the number of concatenated codewords. We can observe that concatenating codewords deteriorates the BER performance. A reasonable BER performance can only be achieved for diffusion-based channels with drift, if a repetition code is applied.

It is important to note that the superior BER performance of the repetition code compared to the ISI-free and DHW code is due to its low code rate. We included the results for the repetition code to show that especially for diffusion-based channels without drift, codes with low rates are required to enable a reliable transmission.

VI. CONCLUSIONS

In this work, we investigated how transposition errors in diffusion-based channels affect the performance of molecular communication. For the first time, we provided a comparative error performance analysis for diffusion-based channels with and without drift. For the uncoded case, we derived an analytical expression for the bit error probability and the simulation results revealed that a reliable transmission is only possible with drift in the medium. To improve the error performance, we applied several block coding techniques and introduced a guard time between the codewords. We observed that for diffusion-based channels with drift a short guard time already improves the performance significantly, whereas diffusion-based channels without drift require a very long guard time. Thus, a reliable transmission over pure diffusion-based channels is only possible when a limited number of bits are sent very rarely (e.g., nano-scale sensor). This research provides the groundwork for future development of strategies to combat transposition errors in pure diffusion-based channels. One possible approach to reduce the long guard time, while having the same error performance, would be to employ enzymes [31].

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